

Combining Probes of Large-Scale Structure

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ABSTRACT

Developing accurate analysis techniques to combine various probes of cosmology is essential to tighten constraints on cosmological parameters and to check for inconsistencies in our model of the Universe.

In this paper we develop a joint analysis framework for six different second-order statistics of tracers of the dark matter density field, namely galaxy position, shear, and magnification. We extend a data compression scheme developed in the context of shear-shear statistics (the so-called COSEBIs) to the other five second-order statistics, thereby significantly reducing the number of data points in the joint data vector.

We simulate a likelihood analysis for the Dark Energy Survey (DES) in a five dimensional cosmological parameter space (plus basic parametrization for bias and correlation parameter) comparing the information content of the individual probes to several combined probes (CP) data vectors. Given the significant correlations of these second-order statistics we model all cross terms in the covariance matrix; furthermore we go beyond the Gaussian covariance approximation and use the halo model to include higher order correlations of the density field.

We find that modeling both, Non-Gaussianity and cross terms, is essential for accurate likelihood contours from the CP data vector; assuming Gaussian errors or uncorrelated probes impacts the contours at a similar level as assuming perfect knowledge of galaxy bias.

We also identify several nulltests based on the degeneracy of magnification and shear statistics which can be used to quantify the contamination of data sets by astrophysical systematics and/or calibration issues.

Key words: cosmology – large scale structure – weak lensing — theory

1 INTRODUCTION

High quality data sets from near-term wide-field imaging surveys, e.g. Kilo-Degree Survey (KiDS¹), Hyper Suprime Cam (HSC²), Dark Energy Survey (DES³), allow for tight constraints on cosmological parameters from the Large Scale Structure (LSS) of the Universe, being complementary with Cosmic Microwave Background (CMB) constraints from the Wilkinson Microwave Anisotropy Probe (WMAP⁴) (see Hinshaw et al. 2012, and references therein) and Planck⁵. The improved data quality and the small statistical uncertainties (as a result of the increased survey volume) pose new challenges for the data analysis; the development and refinement of LSS data analysis methods is crucial for the success of even

larger, future data sets from the Large Synoptic Survey Telescope (LSST⁶), and from future satellite missions Euclid⁷ and the Wide-Field Infrared Survey Telescope (WFIRST⁸).

For all of the aforementioned surveys the tightest constraints on cosmology will be obtained from a joint analysis of all probes that can be extracted from the data (e.g., cluster mass function, shear peak statistics, BAO peak fitting, and various second-order statistics derived from clustering, shear, and magnification). Combining LSS with Supernovae and CMB constraints is straightforward; due to the fact that these probes have very little correlation a joint likelihood analysis frequently comes down to multiplying the corresponding posteriors probabilities (e.g., Kilbinger et al. 2012). Combining the various probes of LSS themselves is complicated for several reasons: First, the cosmological information of various LSS probes is highly correlated, which prohibits a joint analysis

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¹ <http://www.astro.wise.org/projects/KIDS/>

² <http://www.naoj.org/Projects/HSC/HSCProject.html>

³ www.darkenergysurvey.org/

⁴ <http://map.gsfc.nasa.gov/>

⁵ http://www.esa.int/Our_Activities/Space_Science/Planck

⁶ <http://www.lsst.org/lsst>

⁷ sci.esa.int/euclid/

⁸ <http://wfirst.gsfc.nasa.gov/>

on the level of posterior probabilities. Instead the analysis requires a joint likelihood using a covariance matrix that includes all cross correlation terms between the individual probes. Second, not only is the cosmological information correlated, even more problematic are the correlations of various systematic effects originating from astrophysics and the measurements themselves.

As a consequence we do not only have to include the covariance matrix's cross terms in the likelihood analysis, moreover a joint analysis requires a joint modeling of the CP data vector, or more precisely, it requires a prediction code for the combined data vector that ensures consistent modeling of the considered quantities as a function of cosmology and also as a function of the uncertainties in the nuisance parameters. Developing such a CP prediction code is challenging given that modeling the individual probes already requires refined knowledge and high-level expertise on the corresponding astrophysics and systematics. Although this knowledge is present in the corresponding communities, even the individual analysis methods are under constant development in order to meet the new data quality, and unfortunately these methods are largely independent from each other. Phrasing the problem differently: the large correlation of the LSS probes is not reflected in the correlation of the development of the individual analysis techniques.

For example, probably the most important astrophysical uncertainty for clustering based measurements is the relation of dark and luminous matter, modeled through various bias parametrizations and/or Halo Occupation Density (HOD) models. Constraints on these models come from measuring cross correlations of shear and clustering (sometimes called galaxy-galaxy lensing). Cosmic shear uses the same cross terms to offset uncertainties due to intrinsic alignment; simply combining both methods uses the galaxy-galaxy lensing information twice. Similar problems occur when modeling shear calibration which affects cluster masses calibrated through weak lensing, cosmic shear, galaxy-galaxy lensing, and shear peak statistics, all at the same time. Other examples are the modeling of baryonic uncertainties and photo-z calibration which affect all probes but in different ways.

Whereas solving all these problems is beyond the scope of this paper, it is our intention nevertheless to take first steps towards a coherent analysis framework of LSS probes. We limit our problem to second-order statistics (power spectra, correlation functions or linear transformations thereof) that can be derived from a galaxy catalog containing measurements of galaxy shear, galaxy position and galaxy magnification. As we explain further in Sect. 2 we obtain six different second-order statistics from these measurements corresponding to six different probes of the density field. We exclude galaxy clusters and shear peak statistics for now, since these are first-order number count statistics which cause additional complications in the sense that they require a different likelihood function (Poisson distribution instead of Gaussian) in their analysis. Incorporating these probes at the level of the covariance matrix in a Gaussian likelihood together with second-order statistics is questionable.

In terms of nuisance parameters we restrict our analysis to a simple two parameter bias model for probes that involve clustering (see Sect. 5.1), however the extension to other astrophysical contaminations, e.g. intrinsic alignment and baryonic effects is straightforward, and although not being part of the analysis we address it in the discussion.

Within the aforementioned restrictions our analysis advances the existing state of the art of simulated likelihood analysis: 1) We simulate an actual likelihood analysis in a five dimensional cosmological parameter space (and two nuisance parameters), 2) we use a

full Non-Gaussian covariance which includes correlations between all probes and account for higher-order correlations of the density field (see Sect. 4), and 3) we develop a data compression scheme for the joint likelihood analysis which simultaneously solves the shear-shear EB-mode problem.

This data compression scheme was developed by Schneider, Eifler & Krause (2010) (hereafter SEK10) to solve the problem of calculating the a shear E-mode two-point statistics (which contains the cosmological information) from a given shear-shear correlation function on a finite interval. Since information from shear data is limited to angular scales $[\vartheta_{\min}; \vartheta_{\max}]$ any E/B-mode statistic which requires information on larger or smaller scales suffers from so-called E/B-mode mixing or leakage. This problem is examined in Kilbinger, Schneider & Eifler (2006) for configuration space quantities, finding a significant (scale-dependent) bias for formerly used shear statistics, e.g. aperture mass dispersion or shear dispersion. The issue has been addressed in even greater detail for Fourier space quantities, mostly in the context of CMB polarization experiments; several groups developed and refined a Pseudo-Cl technique (Hivon et al. 2002; Brown, Castro & Taylor 2005) that has been applied to simulated shear data in Hikage et al. (2011). In Fourier space E/B-leakage largely depends on the mask of the survey; several mitigation schemes have been developed (Lewis 2003; Smith 2006; Kim & Naselsky 2010).

Except for shear-shear none of the other five second-order statistics suffers from the EB-mode problem; nevertheless the data compression aspects of the COSEBIs are highly desirable for these probes as well. Furthermore, the extension of the COSEBIs scheme allows for a joint cosmological analysis that involves a clean separation of the cosmic shear signal into E- and B-modes.

2 BASIC CONCEPTS

We consider the observables shear γ , magnification μ , and galaxy position g . From these observables the following second-order statistics can be obtained: shear-shear ($\gamma\gamma$), magnification-magnification ($\mu\mu$), galaxy position-galaxy position (gg), shear-magnification ($\gamma\mu$), shear-position (γg), and magnification-position (μg). We want to comprise this second-order cosmological information into a COSEBIs data vector

$$\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\gamma g}, \mathbf{E}^{gg}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\mu g})^t, \quad (1)$$

where each \mathbf{E}^{xx} contains five COSEBI modes (see SEK10, Eifler 2011; Asgari, Schneider & Simon 2012, for justification of the number of modes). The goal of this paper is to simulate a multidimensional likelihood analysis, where “simulated” means that \mathbf{E} is computed from a fiducial cosmological model (see Table 1) using our prediction code; we will refer to this data vector as the *fiducial data vector* from now on.

We assume that the errors of the input data vector \mathbf{E} are described by a multivariate Gaussian

$$L(\mathbf{E}|\mathbf{p}_{\text{co}}) = \frac{1}{(2\pi)^{N/2} \sqrt{|\mathbf{C}|}} \exp \left[-\frac{1}{2} (\mathbf{E} - \mathbf{M})^t \mathbf{C}^{-1} (\mathbf{E} - \mathbf{M}) \right], \quad (2)$$

where \mathbf{p}_{co} denotes the cosmological parameter vector that is assumed in the model vector \mathbf{M} , hence $\mathbf{M} \equiv \mathbf{M}(\mathbf{p}_{\text{co}})$.

The posterior probability in cosmological parameter space is obtained via Bayes' theorem

$$P(\mathbf{p}_{\text{co}}|\mathbf{E}) = \frac{P(\mathbf{p}_{\text{co}}) L(\mathbf{E}|\mathbf{p}_{\text{co}})}{P(\mathbf{E})}, \quad (3)$$

where $P(\mathbf{p}_{\text{co}})$ denotes the prior probability (we assume non-informative priors) and the evidence $P(\mathbf{E})$ can be calculated as an integral over the likelihood $P(\mathbf{E}) = \int d\mathbf{p}_{\text{co}} P(\mathbf{p}_{\text{co}}) L(\mathbf{E}|\mathbf{p}_{\text{co}})$ providing a normalization constant for the posterior probability. Given the functional form of the likelihood as in Eq. (2) the error bars are fully determined by the covariance of the COSEBIs' data vector, which correspondingly to the definition in Eq. (1) reads

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}^{\gamma\gamma\gamma} & \mathbf{C}^{\gamma\gamma g} & \mathbf{C}^{\gamma\gamma\mu} & \mathbf{C}^{\gamma\gamma\mu} & \mathbf{C}^{\gamma\gamma\mu} & \mathbf{C}^{\gamma\gamma\mu} \\ \mathbf{C}^{\gamma g\gamma} & \mathbf{C}^{\gamma g g} & \mathbf{C}^{\gamma g\mu} & \mathbf{C}^{\gamma g\mu} & \mathbf{C}^{\gamma g\mu} & \mathbf{C}^{\gamma g\mu} \\ \mathbf{C}^{\mu\mu\gamma} & \mathbf{C}^{\mu\mu g} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} \\ \mathbf{C}^{\mu\gamma\gamma} & \mathbf{C}^{\mu\gamma g} & \mathbf{C}^{\mu\gamma\mu} & \mathbf{C}^{\mu\gamma\mu} & \mathbf{C}^{\mu\gamma\mu} & \mathbf{C}^{\mu\gamma\mu} \\ \mathbf{C}^{\mu g\gamma} & \mathbf{C}^{\mu g g} & \mathbf{C}^{\mu g\mu} & \mathbf{C}^{\mu g\mu} & \mathbf{C}^{\mu g\mu} & \mathbf{C}^{\mu g\mu} \\ \mathbf{C}^{\mu\mu\gamma} & \mathbf{C}^{\mu\mu g} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} & \mathbf{C}^{\mu\mu\mu} \end{pmatrix}, \quad (4)$$

with \mathbf{C} being symmetric. While postponing a detailed description of the covariance's modeling to Sect. 4, we note that we assume the covariance to be constant with respect to the point in parameter space where the likelihood is evaluated. As shown in Eifler, Schneider & Hartlap (2009) (for Gaussian shear-shear covariances) this assumption is problematic and, depending on the survey parameters, can have significant impact on the parameter constraints. We acknowledge that the covariance matrix, since predicted from a cosmological model, in principle has to vary with respect to cosmology (see Jee et al. 2012; Kilbinger et al. 2012, for corresponding application to shear data) and we will pursue a corresponding extension of this work in the future.

In practice the COSEBIs are calculated from the correlation functions of the three observables. The corresponding power spectra are related to these correlation functions as

$$\xi_{\pm}^{\gamma\gamma}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_{0/4}(\ell\vartheta) C^{\gamma\gamma}(\ell), \quad (5)$$

$$\xi^{\mu\mu}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_0(\ell\vartheta) C^{\mu\mu}(\ell), \quad (6)$$

$$\xi^{gg}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_0(\ell\vartheta) C^{gg}(\ell), \quad (7)$$

$$\xi^{\gamma\mu}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_2(\ell\vartheta) C^{\gamma\mu}(\ell), \quad (8)$$

$$\xi^{\gamma g}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_2(\ell\vartheta) C^{\gamma g}(\ell), \quad (9)$$

$$\xi^{\mu g}(\vartheta) = \frac{1}{2\pi} \int d\ell \ell J_0(\ell\vartheta) C^{\mu g}(\ell), \quad (10)$$

where we point out the J_2 in the polar-scalar correlation functions $\xi^{\gamma\mu}$ and $\xi^{\gamma g}$ (see e.g. Bartelmann & Schneider 2001, for a derivation). We will return to the filter functions in Sect. 5.2.

3 MODELING THE DATA VECTOR

In this section we describe the prediction code for the model data vector (please see Fig. 1 for an illustration), which is an extended version of the shear-shear prediction code described in Eifler (2011). All projected quantities are computed from the non-linear density power spectrum which we calculate from an initial power spectrum $P_{\delta\delta}(k) \propto k^{n_s}$ using the transfer function of Eisenstein & Hu (1998). In order to model the non-linear evolution of the density field we develop a Hybrid approach combining information from Halofit (Smith et al. 2003) and the Coyote Universe Emulator (Lawrence et al. 2010). The latter emulates $P_{\delta\delta}$ over the range $k \in [0.002; 3.4] \text{ h/Mpc}$ within $z \in [0; 1]$ to an accuracy of 1% for cosmologies within $\Omega_m h^2 \in [0.120; 0.155]$, $\Omega_b h^2 \in [0.015; 0.0235]$,

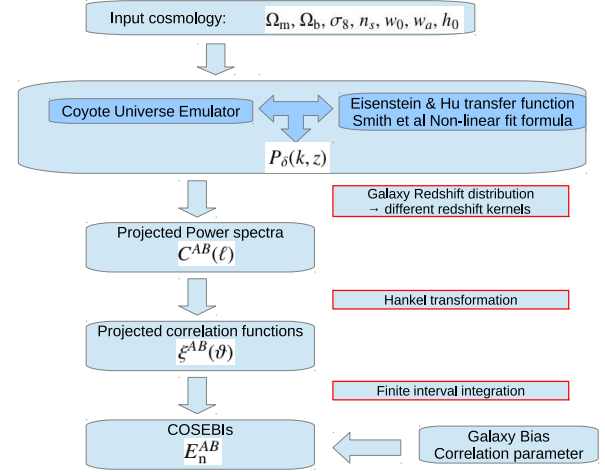


Figure 1. Schematic illustration of the modeling of the CP COSEBIs data vector for a given cosmology.

$n_s \in [0.85; 1.05]$, $\sigma_8 \in [0.6; 0.9]$, $w_0 \in [-1.3; -0.7]$. For any cosmology, z , and k within the aforementioned range, we solely rely on the output of the emulator. For all other parameters we compute the non-linear part of $P_{\delta\delta}$ from Halofit and rescale this solution by a factor

$$f(k, z, \mathbf{p}_{\text{co}}) = \frac{P_{\delta\delta}^{\text{Coyote}}(k, z, \mathbf{p}_{\text{co}}^{\text{close}})}{P_{\delta\delta}^{\text{Halofit}}(k, z, \mathbf{p}_{\text{co}}^{\text{close}})} \quad (11)$$

where \mathbf{p}_{co} is the cosmology parameter vector of interest and $\mathbf{p}_{\text{co}}^{\text{close}}$ is the closest point in parameter space where the Emulator returns a solution (close is defined as minimum difference in each parameter separately).

In order to simulate Λ CDM models we follow the strategy outlined in *icosmo* (Refregier et al. 2011), which interpolates Halofit between flat and open cosmological models to mimic Quintessence cosmologies (please also see Schrabback et al. 2010, for more details). Outside the parameter range of the Emulator the precision of $P_{\delta\delta}$ will of course be significantly below 1%; we nevertheless believe that our approach supersedes other implementations of modeling non-linear structure growth for multiple cosmologies. For example, when using Halofit alone it has been shown that the lensing power spectrum is substantially underestimated (e.g., Hilbert et al. 2009). Throughout this paper we assume a redshift distribution as expected from DES. More precisely, this is modeled by modifying a redshift distribution measured from the Canada-France-Hawaii Telescope Legacy Survey (see Benjamin et al. 2007, adjusted for the lower mean redshift of DES). The exact parameterization reads

$$n(z) = N \left(\frac{z}{z_0} \right)^\alpha \exp \left[- \left(\frac{z}{z_0} \right)^\beta \right], \quad (12)$$

with $\alpha = 2.0$, $\beta = 1.0$, $z_0 = 0.5$.

3.1 Modeling the projected power spectra

Kaiser (1992, 1998) show that projected power spectra are related to the 3D power spectrum of density fluctuations $P_{\delta\delta}$ via a Fourier equivalent of Limber's equation, i.e.

$$P_{12}(\ell) = \int d\chi \frac{q_1(\chi) q_2(\chi)}{f_k^2(\chi)} P_{\delta\delta}(k, \chi) \quad (13)$$

with q_1, q_2 being weight functions, $k = \ell/\chi$, and $f_k(\chi)$ being the comoving angular diameter distance which corresponds to the comoving coordinate χ for the case of vanishing curvature. For simplicity, we will assume the latter in our analysis; note that the tools described in this paper are nevertheless independent of this assumption.

In case of the shear the weight functions q read

$$q_i = \frac{3H_0^2 \Omega_m}{2c^2} \frac{g_i(\chi)\chi}{a} = \frac{3H_0^2 \Omega_m}{2c^2} \frac{\chi}{a} \int_{\chi}^{\chi_h} d\chi' p_i(\chi') \frac{\chi' - \chi}{\chi'}, \quad (14)$$

where $a(\chi)$ is the scale factor and $p_i(\chi)d\chi = p_i(z)dz$ is the redshift distribution of source galaxies in the i^{th} tomography bin. We do not consider tomography in this paper and drop the corresponding denotation of different redshift bins from now on.

Using these weight functions the expression for the shear power spectrum reads

$$C^{\gamma\gamma}(\ell) = \frac{9}{4} \left(\frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_h} d\chi \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi). \quad (15)$$

In the weak lensing approximation the magnification μ equals twice the convergence κ , where the latter equals the shear γ at the level of two-point statistics, hence we can express $C^{\mu\mu} = 4 C^{\kappa\kappa} = 4 C^{\gamma\gamma}$ (see e.g., Bartelmann & Schneider 2001) and subsequently

$$C^{\mu\mu}(\ell) = 9 \left(\frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_h} d\chi \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi). \quad (16)$$

In case of the angular galaxy number density power spectrum the weight function reads $q_i = p_i(\chi) b$ and subsequently we obtain

$$C^{gg}(\ell) = \int_0^{\chi_h} d\chi \frac{p^2(\chi)}{\chi^2} b^2 P_{\delta\delta}(k, \chi). \quad (17)$$

We will specify the bias parameter b further in Sect. 5.

3.2 Projected cross correlation power spectra

Next we consider the cross correlation power spectra between our observables starting with the shear-magnification power spectrum

$$C^{\gamma\mu}(\ell) = \frac{9}{2} \left(\frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_h} d\chi \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi). \quad (18)$$

The corresponding relation for the shear-galaxy position power spectrum reads

$$C^{\gamma g}(\ell) = \frac{3}{2} \left(\frac{H_0}{c} \right)^2 \Omega_m \int_0^{\chi_h} d\chi \frac{g(\chi) p(\chi)}{a(\chi) \chi} b r P_{\delta\delta}(k, \chi), \quad (19)$$

where r denotes the correlation parameter for which, similar to the bias, we postpone an exact description to Sect. 5.

Finally, we obtain

$$C^{\mu g}(\ell) = \frac{3 H_0^2}{c^2} \Omega_m \int_0^{\chi_h} d\chi \frac{g(\chi) p(\chi)}{a(\chi) \chi} b r P_{\delta\delta}(k, \chi) \quad (20)$$

as the expression for the magnification - galaxy position power spectrum.

We note the following interesting relations, which occur as a consequence of the polar-scalar filter functions J_2

$$C^{\mu\mu}(\ell) = 4 C^{\gamma\gamma}(\ell) = 2 C^{\gamma\mu}(\ell) \quad (21)$$

$$\xi^{\mu\mu}(\vartheta) = 4 \xi^{\gamma\gamma}(\vartheta) \neq 2 \xi^{\gamma\mu}(\vartheta) \quad (22)$$

and

$$C^{\mu g}(\ell) = 2 C^{\gamma g}(\ell) \quad (23)$$

$$\xi^{\mu g}(\vartheta) \neq 2 \xi^{\gamma g}(\vartheta). \quad (24)$$

We note that these relations can be used to create linear combinations that can assess the impact of systematics on individual probes. We will expand on this in Sect. 5.2.

3.3 COSEBIs formalism

The COSEBIs' formalism was developed in SEK10; we refer the reader to this paper for details beyond the brief summary presented in this section.

Throughout this paper we only consider filter functions that are logarithmic in ϑ as these filter functions comprise the second order shear information into significantly fewer COSEBI-modes compared to filter functions that are linear in ϑ .

3.3.1 Weak lensing

The COSEBIs shear E-mode, denoted as E_n , can be expressed as an integral over the shear 2PCF ξ_{\pm} as

$$E_n^{\gamma\gamma} = \frac{1}{2} \int_{\vartheta_{\min}}^{\vartheta_{\max}} d\vartheta \vartheta [T_{n+}^{\gamma\gamma}(\vartheta) \xi_+^{\gamma\gamma}(\vartheta) + T_{n-}^{\gamma\gamma}(\vartheta) \xi_-^{\gamma\gamma}(\vartheta)]. \quad (25)$$

Note that for a properly constructed $T_{n+}^{\gamma\gamma}$ as described below the corresponding $T_{n-}^{\gamma\gamma}$ can be readily calculated as

$$T_{n-}^{\gamma\gamma}(\vartheta) = T_{n+}^{\gamma\gamma}(\vartheta) + \int_0^{\vartheta} d\theta \theta T_{n+}^{\gamma\gamma}(\theta) \left(\frac{4}{\vartheta^2} - \frac{12\theta^2}{\vartheta^4} \right). \quad (26)$$

For further details on this the reader is referred to SEK10 and references therein. In the following we only describe the construction of $T_{n+}^{\gamma\gamma}$.

In order to allow for a proper E/B-modes separation using a 2PCF over only a finite interval the filter functions $T_{n+}^{\gamma\gamma}$ must meet the requirement

$$\int d\vartheta \vartheta T_{n+}^{\gamma\gamma}(\vartheta) = 0 = \int d\vartheta \vartheta^3 T_{n+}^{\gamma\gamma}(\vartheta). \quad (27)$$

In addition, the set of filter functions $T_{n+}^{\gamma\gamma}$ must be orthonormal, i.e.

$$\frac{1}{\Delta\vartheta} \int_{\vartheta_{\min}}^{\vartheta_{\max}} d\vartheta T_{n+}^{\gamma\gamma}(\vartheta) T_{m+}^{\gamma\gamma}(\vartheta) = \delta_{mn}. \quad (28)$$

The explicit construction of the logarithmic T_{n+} is described in SEK10. The main steps of the construction are:

- A variable transformation $\vartheta \rightarrow z = \ln(\vartheta/\vartheta_{\min})$
- Expressing Eqs. (27, 28) in z with $T_{n+}^{\gamma\gamma}(\vartheta) \rightarrow t_{n+}^{\gamma\gamma}(z)$
- Expanding each $t_{n+}^{\gamma\gamma}(z) = \sum_{j=0}^{n+1} c_{nj} z^j$
- Calculating the coefficients c_{nj} from the conditions (27, 28)

Given n , the filter function $T_{n+}^{\gamma\gamma}$ will be of order $n+1$ as it needs to fulfill $n+1$ constraints, i.e. it must fulfill Eq. (28) for all $T_{m+}^{\gamma\gamma}$ with $m \leq n-1$ and additionally it has to meet the two EB-mode separation constraints in Eq. (27). This implies that $T_{1+}^{\gamma\gamma}$ is of order two.

3.3.2 Extension to clustering and magnification

Having determined the weak lensing COSEBIs filter functions $T_{n+}^{\gamma\gamma}$, we can calculate the other five probe's COSEBIs similar to Eq. (25)

$$E_n^{AB} = \int_{\vartheta_{\min}}^{\vartheta_{\max}} d\vartheta \vartheta T_n^{AB}(\vartheta) \xi^{AB}(\vartheta). \quad (29)$$

In this paper we assume the same configuration space filter function $T_{n+}^{\gamma\gamma}$ for all other probes, henceforth neglecting the superscripts.

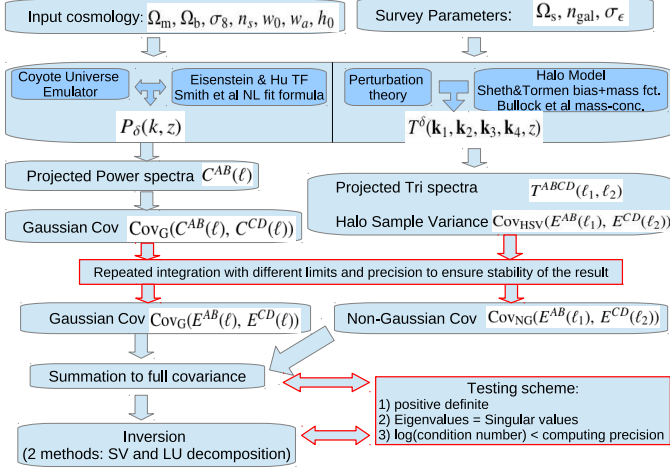


Figure 2. Schematic illustration of the modeling of the joint COSEBIs covariance for a given cosmology.

In addition to Eq. (29) the COSEBIs can be calculated directly from the power spectrum

$$E_n^{AB} = \frac{1}{2\pi} \int d\ell \ell W_n^{AB}(\ell) C^{AB}(\ell), \quad (30)$$

which we only use as a consistency check since computing $C^{AB}(\ell) \rightarrow \xi^{AB}(\theta)$ using a fast Hankel-transformation and subsequently carrying out the finite integration in Eq. (29) is significantly faster.

The Fourier filter functions $W_n^{AB}(\ell)$ are needed however for the computation of the COSEBI's covariance; they can be obtained from the T_n as

$$W_n^{\gamma\gamma}(\ell) = W_n^{\mu\mu}(\ell) = W_n^{gg}(\ell) = W_n^{\mu g}(\ell) = \int d\vartheta \vartheta T_n(\vartheta) J_0(\ell\vartheta), \quad (31)$$

$$W_n^{\gamma g}(\ell) = W_n^{\gamma\mu}(\ell) = \int d\vartheta \vartheta T_n(\vartheta) J_2(\ell\vartheta), \quad (32)$$

where the $J_{0/2}$ are a consequence of Eqs. (5 - 10).

4 MODELING OF COVARIANCES

We first describe the modeling of covariances for projected power spectra; the expression for computing the COSEBIs covariance from the power spectrum covariance is straightforward, however the actual computation is easily affected by numerical uncertainties. We outline our method and cross checks at the end of this section.

4.1 Power spectrum covariances

Under the assumption that the density field is Gaussian (which means that the shear four-point function can be expressed in terms of two-point functions) the covariance of projected shear power spectra can be expressed as (Hu & Jain 2004)

$$\begin{aligned} \text{Cov}_G(C^{AB}(\ell)C^{CD}(\ell')) &= \langle \Delta C^{AB}(\ell) \Delta C^{CD}(\ell') \rangle \\ &= \frac{2\pi \delta_{\ell\ell'}}{\Omega_s \ell \Delta \ell} \left[(C^{AC}(\ell) + N^{AC})(C^{BD}(\ell) + N^{BD}) \right. \\ &\quad \left. + (C^{AD}(\ell) + N^{AD})(C^{BC}(\ell) + N^{BC}) \right], \end{aligned} \quad (33)$$

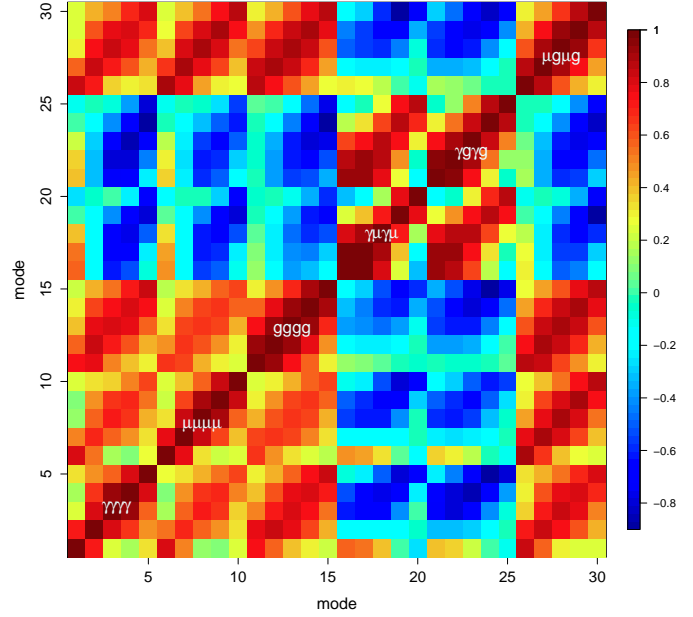


Figure 3. Full (Gaussian+Non-Gaussian) COSEBIs correlation matrix. Since we assume five modes for each of the six probes our data vector contains 30 data points, hence the covariance is 30×30 . We indicate the corresponding auto-covariance block matrices in the plots.

where the superscripts are to be replaced with γ, g, μ depending on the probe under consideration and Ω_s denotes the survey volume. The covariance gets contributions from the signal $C(\ell)$ and a noise term N . Note that

$$N_{\gamma\gamma} = \frac{\sigma_\epsilon^2}{2n_{\text{gal}}}, \quad N_{\mu\mu} = \frac{\sigma_\mu^2}{n_{\text{gal}}}, \quad N_{gg} = \frac{1}{n_{\text{gal}}}, \quad (34)$$

and all other noise terms are zero. We assume the intrinsic shape noise, $\sigma_\epsilon^2 = 0.32$, and note that the factor “2” in the denominator results from the fact that the shear has two components. For the magnification noise parameter we follow the arguments in Krause et al. (2013) defining $\sigma_\mu = 2\sigma_\kappa/f^{1/2}$ for scaling relation based estimators of magnification (Huff & Graves 2011), where f denotes the fraction of galaxies for which magnification is measured and σ_κ being the scatter of the convergence estimator. Being optimistic that the method in Huff & Graves (2011) can be extended to late type galaxies we assume $\sigma_\mu = 1.2$, noting that the uncertainty of this noise level is large.

Since non-linear structure growth at late time induces significant non-Gaussianities in the density field, Eq. (33) underestimates the error on cosmological parameters and needs to be amended by an additional term, i.e. $\text{Cov} = \text{Cov}_G + \text{Cov}_{\text{NG}}$. We model non-Gaussian covariance as the sum of the trispectrum contributions (Cooray & Hu 2001; Takada & Jain 2009), including a sample variance term which describes the scatter in power spectrum measurements due to large scale density modes (Takada & Bridle 2007; Sato et al. 2009),

$$\begin{aligned} \text{Cov}_{\text{NG}}(C^{AB}(l_1), C^{CD}(l_2)) &= \frac{1}{\Omega_s} \int_{|\mathbf{l}| \in l_1} \frac{d^2 \mathbf{l}}{A(l_1)} \\ &\quad \times \int_{|\mathbf{l}'| \in l_2} \frac{d^2 \mathbf{l}'}{A(l_2)} T^{ABCD}(\mathbf{l}, -\mathbf{l}, \mathbf{l}', -\mathbf{l}') \end{aligned} \quad (35)$$

with $T^{ABCD}(\mathbf{l}, -\mathbf{l}, \mathbf{l}', -\mathbf{l}')$ defined as

$$T^{\gamma\alpha\mu\beta g^{4-\alpha-\beta}}(l_1, l_2) = 2^\beta \left(\frac{3}{2} \frac{H_0^2}{c^2} \Omega_m \right)^{\alpha+\beta} \int_0^{\chi_h} d\chi \left(\frac{g(\chi)\chi}{a(\chi)} \right)^{\alpha+\beta} \times (p(\chi)b)^{4-\alpha-\beta} \chi^{-6} T^{\delta\delta\delta\delta} \left(\frac{l_1}{\chi}, \frac{l_2}{\chi}, \chi \right), \quad (36)$$

where we assume the correlation parameter $r = 1$ and $\alpha, \beta \in [0; 4]$. For example the pure shear tri-spectrum $T^{\gamma\gamma\gamma\gamma}$, and the pure galaxy position tri-spectrum T^{gggg} read

$$T^{\gamma^4}(l_1, l_2) = \left(\frac{3}{2} \frac{H_0^2}{c^2} \Omega_m \right)^4 \int_0^{\chi_h} d\chi \frac{g^4(\chi)}{a^4(\chi)\chi^2} T^{\delta\delta\delta\delta} \left(\frac{l_1}{\chi}, \frac{l_2}{\chi}, \chi \right), \quad (37)$$

$$T^{g^4}(l_1, l_2) = \int_0^{\chi_h} d\chi \frac{p^4(\chi)}{\chi^6} T^{\delta\delta\delta\delta} \left(\frac{l_1}{\chi}, \frac{l_2}{\chi}, \chi \right), \quad (38)$$

respectively.

4.2 Halo Model Trispectrum

We model the dark matter trispectrum using the halo model (Seljak 2000; Cooray & Sheth 2002), which assumes that all matter is bound in virialized structures that are modeled as biased tracers of the density field. Within this model the statistics of the density field can be described by the dark matter distribution within halos on small scales, and is dominated by the clustering properties of halos and their abundance on large scales. In this model, the trispectrum splits into five terms describing the 4-point correlation within one halo (the *one-halo* term T^{1h}), between 2 to 4 halos (*two-, three-, four-halo* term), and a so-called halo sample variance term T^{HSV} , caused by fluctuations in the number of massive halos within the survey area,

$$T = T_{1h} + (T_{2h,(22)} + T_{2h,(13)}) + T_{3h} + T_{4h} + T^{\text{HSV}}. \quad (39)$$

The *two-halo* term is split into two parts, representing correlations between two or three points in the first halo and two or one point in the second halo. As halos are the building blocks of the density field in the halo approach, we need to choose models for their internal structure, abundance and clustering in order to build a model for the trispectrum. Our implementation of the one-, two- and four-halo term contributions to the matter trispectrum follow (Cooray & Hu 2001), and we neglect the three-halo term as it is subdominant compared to the other terms as the scales of interest for this analysis. Specifically, we assume NFW halo profiles (Navarro, Frenk & White 1997) with the Bullock et al. (2001) fitting formula for the halo mass–concentration relation $c(M, z)$, and the Sheth & Tormen (1999) fit functions for the halo mass function $\frac{dn}{dM}$ and linear halo bias $b(M)$, neglecting terms involving higher order halo biasing.

Table 1. Cosmological parameter ranges used in the likelihood analyses.

parameter	3D analyses	5D analyses	fiducial
Ω_m	[0.18; 0.38]	[0.24; 0.29]	0.2646
σ_8	[0.65; 0.95]	[0.76; 0.84]	0.801
w_0	[-0.5; -2.0]	[-0.8; -1.2]	-1.0
n_s	0.963	[0.85; 1.06]	0.963
w_a	0.0	[-2.0; 2.0]	0.0
b	-	[0.9; 1.3]	1.2
r	-	[0.8; 1.1]	1.0

4.3 COSEBIs covariances

Following Sato et al. (2009) the halo sample variance term can be calculated as

$$\text{Cov}_{\text{HSV}}(C^{AB}(l_1), C^{CD}(l_2)) = 2^\beta \left(\frac{3}{2} \frac{H_0^2}{c^2} \Omega_m \right)^{\alpha+\beta} \times \int_0^{\chi_h} d\chi \left(\frac{d^2 V}{d\chi d\Omega} \right)^2 \left(\frac{g(\chi)\chi}{a(\chi)} \right)^{\alpha+\beta} (p(\chi)b)^{4-\alpha-\beta} \times \int dM \frac{dn}{dM} b(M) \left(\frac{M}{\bar{\rho}} \right)^2 |\tilde{u}(l_1/\chi, c(M, z(\chi)))|^2 \times \int dM' \frac{dn}{dM'} b(M') \left(\frac{M'}{\bar{\rho}} \right)^2 |\tilde{u}(l_1/\chi, c(M', z(\chi)))|^2 \times \int_0^\infty \frac{kd k}{2\pi} P_\delta^{\text{lin}}(k, z(\chi)) |\tilde{W}(k\chi\Theta_s)|^2, \quad (40)$$

with $\tilde{u}(l_1/\chi, c(M, z(\chi)))$ the normalized Fourier transform of the halo density profile.

Adding Gaussian (Eq. 33) and non Gaussian covariance (Eq. 35) and subsequently integrating over ℓ and ℓ' we obtain the final COSEBI's covariance

$$\text{Cov}(E_n^{AB}, E_m^{CD}) = \frac{1}{4\pi^2} \left[\int_0^\infty d\ell \ell W_n^{AB}(\ell) W_m^{CD}(\ell) \text{Cov}_G(\ell) + \int_0^\infty d\ell \ell W_n^{AB}(\ell) \int_0^\infty d\ell' \ell' W_m^{CD}(\ell') \text{Cov}_{\text{NG}}(l_1, l_2) \right]. \quad (41)$$

This integration must be tested thoroughly for convergence and stability of the result with respect to numerical integration precision, upper and lower limit of the integration. If the result is stable we verify that the covariance and its inverse is positive definite. In Fig. 3 we show the correlation matrix of the full covariance matrix. Since the COSEBIs filter functions average over all Fourier modes/angular scales, it is difficult to have an intuitive understanding of this matrix. We show the impact on likelihood contours when neglecting the Non-Gaussian terms in Fig. 5 and when neglecting the cross terms of the matrix in Fig. 6.

5 LIKELIHOOD ANALYSIS

In this section we simulate various likelihood analyses using the data vector in Eq. (1) and covariance in Eq. (4) or subsets thereof for survey parameters close to what is expected for the Dark Energy Survey ($\Omega_s = 5000 \text{ deg}^2$, $n_{\text{gal}} = 10/\text{arcmin}^2$). The range of the cosmological parameter space considered in the different analyses is summarized in Table 1. We calculate the likelihood as described in Eq. (2) and subsequently the posterior probability via Eq. (3). All posteriors are computed on a three- and five-dimensional hypercube, respectively, and then projected onto two dimensions. The

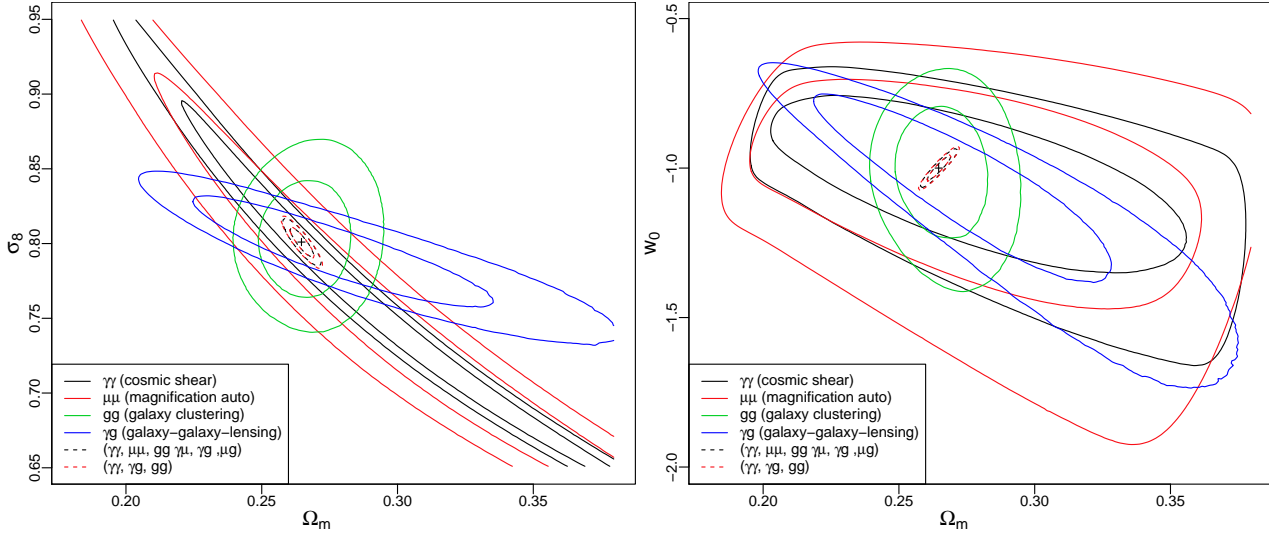


Figure 4. Likelihood analysis in three-dimensional cosmological parameter space (*left*: Ω_m vs σ_8 marginalized over w_0 , *right*: Ω_m vs w_0 , marginalized over σ_8). We show the 68% and 95% credible regions for six different data vectors, the solid lines correspond to constraints when using the single probes individually, the dashed lines result from a joint analysis of shear and clustering data (red) $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\gamma g}, \mathbf{E}^{gg})$, and when using the full six second-order statistics from clustering, shear, and magnification (black) $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{gg}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\gamma g}, \mathbf{E}^{\mu g})$

contour plots in Figs. 4 -7 show the marginalized probability calculated as

$$L(\mathbf{E}|\mathbf{p}_{co}(2D)) = \int_{\mathbf{p}'_{co}(min)}^{\mathbf{p}'_{co}(max)} d\mathbf{p}'_{co} L(\mathbf{E}|\mathbf{p}'_{co})$$

where \mathbf{p}'_{co} denotes the remaining cosmological parameters when subtracting the considered two parameters from the full parameter set.

The schematic illustrations in Fig. 1 and Fig. 2 show the computation of the data vector and covariance, respectively. Note that the COSEBIs data vector is computed from the corresponding correlation function over an interval of $[1'; 400']$. Computing time for the full COSEBIs data vector is <1 sec per cosmology.

Comparing individual and combined probes In our first likelihood analysis we compare the cosmological information of individual probes to a CP analysis. Figure 4 shows constraints from the single probe data vectors $\mathbf{E}^{\gamma\gamma}$, $\mathbf{E}^{\mu\mu}$, and \mathbf{E}^{gg} (black, red, and green contours, respectively) and their corresponding covariance matrices (submatrices of Eq. 4) to the six-probe data vector (dark blue contours) described in Eq. (1) and a three-probe data vector $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\gamma g}, \mathbf{E}^{gg})$ (light blue contours) which contains only information from shear and position. In this analysis we only consider a three dimensional cosmological parameter space (Ω_m , σ_8 , and w_0) for the reason that the individual probes error bars cover a wide range of cosmologies (which broaden even further for higher dimensional parameter spaces), whereas the CP contours require a high resolution of grid points. The combination of high resolution and large parameter space given our limited computing resources and brute-force likelihood approach limits this part of the analysis to 3D.

However, already in the 3D case the results are clear: First, there is significant improvement for the CP data vector over the individual probes and second, the inclusion of magnification as a probe at the assumed noise level does not substantially contribute to the information in this parameter space. The latter result also holds for the five-dimensional likelihood analysis (see Fig. 6) and can be ex-

plained by the fact that the information is highly degenerate with shear.

Comparing Gaussian and Non-Gaussian covariances Compared to Fig. 4, we extend the considered cosmological parameter space in Fig. 5 by the spectral index of the power spectrum n_s . We compare Gaussian and Non-Gaussian covariances and their impact on parameter estimation; for likelihood analyses of individual probes such comparisons have been carried out in previous papers (see e.g. Takada & Jain 2009; Eifler, Schneider & Hartlap 2009, for cosmic shear), this however is the first time that such a comparison 1) is shown for the CP case and 2) includes the Halo Sample Variance term in the covariance. By comparing the relative change in likelihood contours of Fig. 5 and Fig. 7 we find that both effects, i.e. neglecting Non-Gaussianity and assuming perfect knowledge of galaxy bias, have comparable impact on the constraints.

Comparing different CP data vectors Here, we further extend our analysis by one cosmological dimension and include a simple time-dependent dark energy component w_a . For a non-tomographic analysis as carried out in this paper constraints on w_a are rather poor; overall our marginalized constraints will improve substantially when extending our formalism to tomography. Figure 6 compares two different data vectors, namely $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{gg}, \mathbf{E}^{\gamma g}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\mu g})$ (black and blue contours) and $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{gg}, \mathbf{E}^{\gamma g})$ (red contours); in the first case we further compare the full covariance including cross terms amongst probes (black) to the case where we ignore these cross terms and only consider the auto-correlation block matrices in the covariance matrix (blue).

We find a slight but not very significant improvement of the cosmological information when including the magnification terms in the data vector, for the same reasons as outlined in Sect. 5. Using only the block diagonal auto-correlation covariance matrix we see a notable change in the contours, in particular we also find a change in the degeneracies within the cosmological parameter space. Also here, we compare the relative change in contour size in Fig. 6 (blue and black) to the increase in contour size when dropping the as-

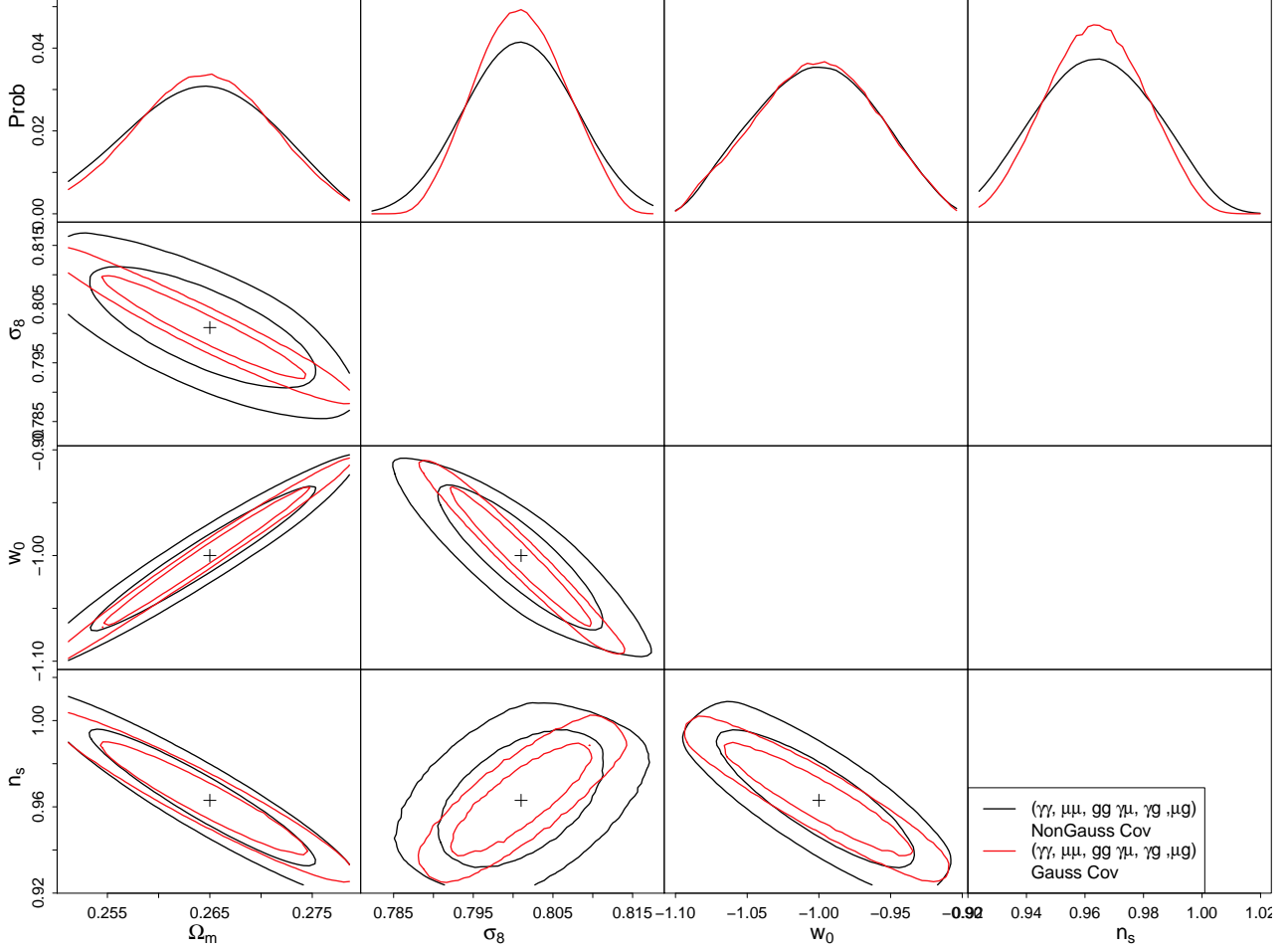


Figure 5. Likelihood analysis in four-dimensional cosmological parameter space. We show the 68% and 95% credible regions and marginalize over the all other parameters not shown in a given panel. The colors correspond to different data vectors and covariances used in the likelihood analysis: *black*: $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{gg}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\mu g})$ data vector and full Non-Gaussian covariance, *red*: $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{gg}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\mu g})$ data vector but using the Gaussian covariance instead.

sumption of perfectly known bias (see Fig. 7). We find that neglecting the cross terms has a comparable impact on the change in contour size as assuming perfect knowledge of galaxy bias.

5.1 Uncertainties from Bias and Correlation Parameters

Understanding the relation of galaxies and their dark matter environment is an important aspect of any cosmological parameter estimation that includes clustering information. Constraining and modeling this relation is an active field of research in theory (e.g., Zheng et al. 2005; McDonald & Roy 2009) and observations (e.g., Cacciato et al. 2012; Mandelbaum et al. 2012; Jullo et al. 2012). In practice, any bias model will have to be finetuned to the considered data set (galaxy population/morphology and redshift distribution). Guidance on any parametrization from first physical principles is limited; measurements rely mostly on configuration space quantities, i.e. a parametrization in r , z or ϑ .

Bias and correlation parameter can also be parametrized (e.g. Bartelmann & Schneider 2001; Bernstein 2009) as a function of k , z in Eqs. (17, 19), more precisely

$$b^2(\mathbf{k}|\chi) = \frac{P_{gg}(\mathbf{k}|\chi)}{P_{\delta\delta}(\mathbf{k}|\chi)}, \quad (42)$$

and

$$r(\mathbf{k}|\chi) = \frac{P_{\delta g}(\mathbf{k}|\chi)}{\sqrt{P_{\delta\delta}(\mathbf{k}|\chi)P_{gg}(\mathbf{k}|\chi)}}. \quad (43)$$

with P_{gg} being the observable galaxy number density power spectrum and $P_{\delta g}$ being the galaxy-dark matter cross power spectrum. In contrast, we parametrize b and r directly as a function of the quantity that enters the likelihood analysis, i.e. we define the relation between the observable galaxy number density COSEBIs E_n^{gg} and the dark matter density COSEBIs $E_n^{\delta\delta}$ as

$$E_n^{gg} = X_n E_n^{\delta\delta}, \quad (44)$$

and similarly

$$E_n^{\gamma g} = Y_n E_n^{\gamma\delta}. \quad (45)$$

For the case of scale-independent bias and correlation parameter, $X_n = b^2$ and $Y_n = br$ for all modes n .

Scale and redshift dependence of b and r will result in different values of X_n (for different modes n) with a correlation induced by the functional form of the scale dependence (this also holds for Y_n and also for the relation between X_n and Y_n). Below, we address the issue of how to determine these relations further by proposing a measurement scheme to constrain X_n , Y_n .

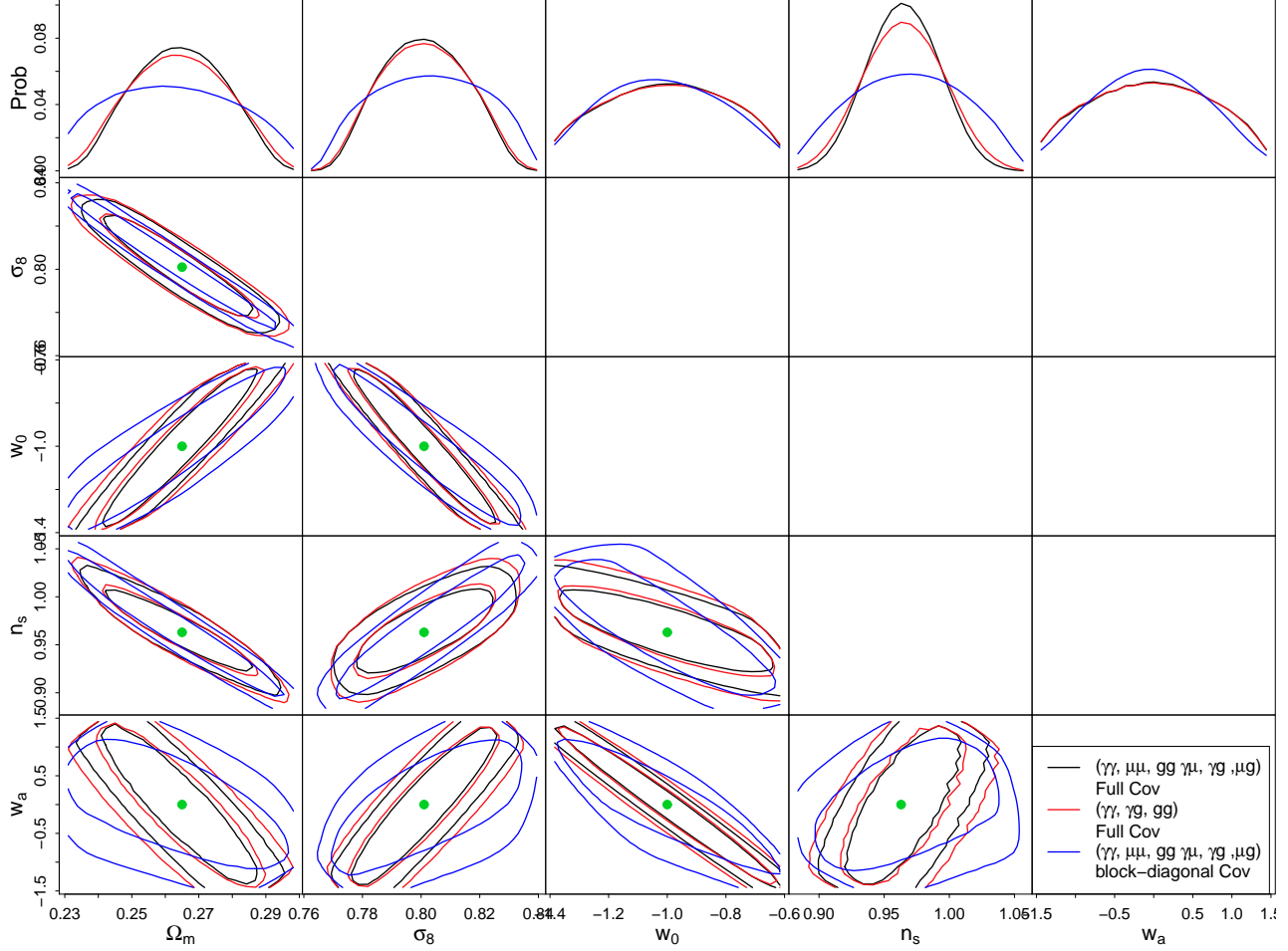


Figure 6. Likelihood analysis in five dimensional cosmological parameter space. We show the 68% and 95% credible regions and marginalize over the all other parameters not shown in a given panel. The colors correspond to different data vectors and covariances used in the likelihood analysis: *black*: $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\gamma g}, \mathbf{E}^{\mu g})$ data vector and full Non-Gaussian covariance, *red*: $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\gamma g}, \mathbf{E}^{\mu g})$ data vector and corresponding Non-Gaussian covariance, *blue*: $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\gamma g}, \mathbf{E}^{\mu g})$ data vector but using only the diagonal block matrices in the covariance.

In this analysis we bracket $X_n \in [b_{\min}^2; b_{\max}^2]$ and $Y_n \in [b_{\min} r_{\min}; b_{\max} r_{\max}]$ with the limits on b, r in Table 1, which were conservatively chosen using guidance from observational results from the Red-sequence Cluster Survey (Fig. 9 in Cacciato et al. 2012). These upper and lower limits are bracketing all X_n, Y_n , even if b and r are functions of k, z .

Figure 7 shows the results of our simulated likelihood analysis in $\Omega_m, \sigma_8, w_0, n_s, w_a$ parameter space assuming perfect knowledge of bias and correlation parameter (red contours), and using the parametrization outlined in Eqs. (44, 45). Our bias parametrization is conservative in the sense that we do not assume any correlation between X_n and Y_n other than that upper and lower limits are calculated from upper and lower limits of b and r .

As expected marginalizing over uncertainties in bias and correlation parameter significantly weakens cosmological constraints, especially for σ_8 .

In the following we suggest a measurement framework to make progress on the brackets of X_n and Y_n observationally. In the context of the aperture mass dispersion this method has been suggested in van Waerbeke (1998) to detect scale dependence of galaxy bias by combining second-order statistics of shear and clustering (see Hoekstra et al. 2002; Cacciato et al. 2012, for application to data).

For the COSEBIs the corresponding relations read

$$X_n = f_X \frac{E_n^{gg}}{E_n^{\gamma\gamma}} \quad (46)$$

and

$$Y_n = f_Y \frac{E_n^{\gamma g}}{\sqrt{E_n^{\gamma\gamma} E_n^{\mu\mu}}}. \quad (47)$$

The functions f_X and f_Y depend weakly on cosmology (Hoekstra et al. 2002; Schneider, Kochanek & Wambsganss 2006), which can be mitigated even further by employing strong priors from independent experiments, e.g. Planck. This method allows to measure and constrain a “mode-dependent” bias for the COSEBIs. We however note that on cosmological scales galaxy bias has little scale but significant redshift dependence and that this method should be applied within sufficiently small tomography bins.

Analogous relations can be derived using $E_n^{\mu\mu}$ instead of $E_n^{\gamma\gamma}$ and/or using $E_n^{\mu g}$ instead of $E_n^{\gamma g}$. We emphasize that magnification can provide additional information to constrain the relation between dark and luminous matter; at the very least it provides a valuable cross check/nulltest for the above method.

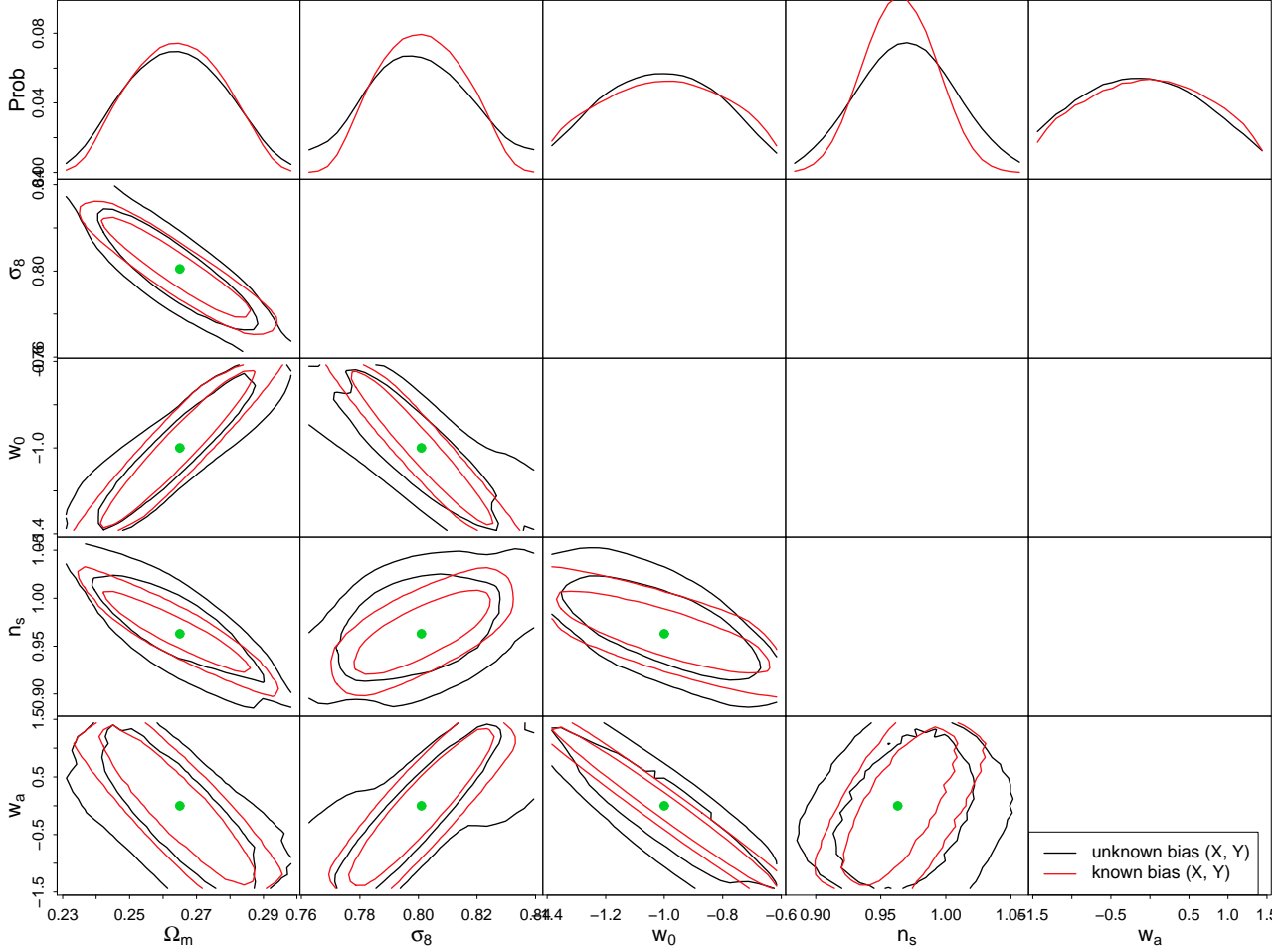


Figure 7. Likelihood analysis in five-dimensional cosmological parameter space using the $\mathbf{E}^{\gamma\gamma}$, $\mathbf{E}^{\mu\mu}$, \mathbf{E}^{gg} , $\mathbf{E}^{\gamma\mu}$, $\mathbf{E}^{\gamma g}$, $\mathbf{E}^{\mu g}$ data vector and the corresponding Non-Gaussian covariance. *Black* contours correspond to a likelihood analysis assuming perfect knowledge of bias and *red* contours correspond to marginalizing over 2 bias parameters $b \in [0.9; 1.3]$ and $r \in [0.8; 1.1]$. We show the 68% and 95% credible regions and marginalize over the all other parameters not shown in a given panel.

5.2 Nulltests involving shear and magnification

The fact that including magnification contributes only little to the cosmological constraints is not unexpected given the degeneracy of shear and magnification. This degeneracy however allows us to test for and to quantify systematics that affect both probes differently. As an example we will assume that one of the most important contaminations of cosmic shear, i.e. intrinsic alignment, does not affect magnification. We can express the observed shear power spectrum as a the sum of the true shear power spectrum and the two intrinsic alignment components II (correlation of the intrinsic ellipticity with the local density field) and GI (correlation of foreground galaxy ellipticity with background shear) (Hirata & Seljak 2004; Bernstein 2009; Joachimi & Bridle 2010)

$$C_{\text{obs}}^{\gamma\gamma}(\ell) = C^{\gamma\gamma}(\ell) + C^{\text{II}}(\ell) + C^{\text{GI}}(\ell), \quad (48)$$

$$C_{\text{obs}}^{\mu\mu}(\ell) = C^{\mu\mu}(\ell). \quad (49)$$

Using Eq. (22) in terms of the COSEBIs we can rewrite Eqs. (48, 49) as

$$E_n^{\text{II}} + E_n^{\text{GI}} = E_n^{\mu\mu}(\text{obs}) - 4 E_n^{\gamma\gamma}(\text{obs}), \quad (50)$$

thereby constraining intrinsic alignment. We note that contaminations similar to IA for shear might exist for magnification as well;

correlations of the local density field with the intrinsic size of the galaxies (II^{μ}), and/or correlations of the foreground galaxy size with the magnification of a background galaxy (GI^{μ}) are likely. As discussed in Schmidt et al. (2012) magnification estimators which are not based on the excess of number densities but on size measurements have little correlation with their environment (e.g. Croton et al. 2005; Maltby et al. 2010) therefore potentially allowing the above technique to be successful.

In any case the above nulltest can be used for other types of contaminations which dominate the shear related quantity but do not affect magnification; the best example probably being shear calibration.

The number of nulltests is not limited to the magnification and shear auto-correlations but additional constraints can be gained from three other relations similar to Eqs. (21, 23). As a prerequisite we define new COSEBIs filter functions for γg and $\gamma\mu$, denoted as T'_n such that

$$W_n^{\gamma\gamma}(\ell) = W_n^{\gamma g}(\ell) = W_n^{\gamma\mu}(\ell), \quad (51)$$

$$\int d\vartheta \vartheta T_n(\vartheta) J_0(\ell\vartheta) = \int d\vartheta \vartheta T'_n(\vartheta) J_2(\ell\vartheta), \quad (52)$$

with T_n still being the original filter function $T_{n+}^{\gamma\gamma}$ defined in Sect. 3.3.1. Calculating the new COSEBIs $E_n^{\gamma g}$ and $E_n^{\gamma\mu}$ as an integral over the corresponding correlation functions $\xi^{\gamma g}$ and $\xi^{\gamma\mu}$ using the T'_n we derive the relations

$$E_n^{\mu\mu} = 2 E_n^{\gamma\mu}, \quad (53)$$

$$E_n^{\gamma\mu} = 2 E_n^{\gamma\gamma}, \quad (54)$$

$$E_n^{\mu g} = 2 E_n^{\gamma g}, \quad (55)$$

which do not hold for the correlation functions (see Eqs. 22, 24). These relations or linear combinations thereof can be used to define nulltests and subsequently constrain astrophysical uncertainties.

6 CONCLUSIONS

In this paper we develop a coherent analysis framework to extract constraints on cosmology from all second-order statistics that can be derived from a galaxy position (g), shear (γ), and magnification (μ) catalog.

For the joint data vector $\mathbf{E} = (\mathbf{E}^{\gamma\gamma}, \mathbf{E}^{\gamma g}, \mathbf{E}^{gg}, \mathbf{E}^{\mu\mu}, \mathbf{E}^{\gamma\mu}, \mathbf{E}^{\mu g})$ we develop a fast prediction code that computes projected quantities quickly from a density power spectrum generated by the Coyote Universe Emulator or a modified Halofit implementation. We also developed code to predict non-Gaussian covariances for all aforementioned projected quantities which is based on the halo model. We then generate a CP data vector from our prediction code assuming a fiducial cosmology and test this data vector (and subsets thereof) in several likelihood analyses, the most extensive one covering five cosmological dimensions $\Omega_m, \sigma_8, w_0, n_s, w_a$ and a two parameter galaxy bias model.

The analysis scheme suggested in this paper differs from previous work in several ways: First, we include all second-order cross statistics of the observables (g, γ, μ) into the data vector (thereby increasing the sources of cosmological information), second, we model all cross terms in the covariance matrix, and third, we include all higher order correlations of the density field in the covariance matrix. Furthermore we employ the COSEBIs formalism to quantify the information content, thereby solving the cosmic shear E/B-mode problem and introducing a data compression scheme for the other five two-point statistics.

Not surprisingly, we find substantial improvement in parameter constraints when using the CP data vector instead of the individual probes, and a sizable increase in the CP likelihood contours when fitting for galaxy bias instead of assuming it perfectly known.

The most interesting result of this paper is the change in likelihood contours when including the Non-Gaussianity of the density field and all cross terms between the individual probes. We find that neglecting either of these features has a comparable impact on likelihood contours as the uncertainty of galaxy bias. This statement of course only applies to the CP case considered in this paper; when only considering galaxy clustering, modeling galaxy bias will still be the dominant uncertainty. We however emphasize that forecasting exercises for a CP analysis similar to ours should incorporate Non-Gaussian covariances including all cross terms.

The inclusion of magnification slightly improves cosmological constraints although at the assumed noise level $\sigma_\mu = 1.2$ this improvement is small. We note that magnification in contrast to shear and clustering is a relatively recent cosmological probe, hence our assumed noise level might be too pessimistic. The complete degeneracy of shear and magnification however allows for interesting constraints on systematics, e.g. intrinsic alignment,

shear calibration, etc. We outline several relations that hold in the absence of these systematics and suggest extensions of these nulltests.

Scientifically interesting is the extension of our framework to other second-order statistics whose distribution follow the same likelihood function, e.g. CMB polarization, CMB lensing, CMB temperature correlations, and of course also the extension to tomography. It is straightforward to apply this analysis scheme to a any data set from which can measure projected correlation functions. Before extracting meaningful information from data however, the framework described here needs extensions. For example, implementing a detailed HOD-model approach (van den Bosch et al. 2012), adding the parametrization of nuisance parameters, such as baryons, intrinsic alignment, photo-z calibration, and shear calibration is required. Although this is challenging infrastructure work, the inclusion of these add-ons poses no fundamental problem for our framework.

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